Deformability of Sands at Plane Strain Condition

Déformation Plane du Sable

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SYNOPSIS The paper describes the results of experimental studies performed from the apparatus with independent application of three principal stresses at different stress paths. Behaviour of sand under plane strain is analysed. All test results are presented in tables and diagrams.

Stress-strain behaviour of the soil in foundations of industrial, civil and hydraulic structures often identifies the plane strain condition. However the strength and deformability properties of the soil used in engineering desings, are usually determined from the results of the triaxial apparatus with no corrections being introduced for the plane strain analysis. The results of investigations by Cornforth (1964), Green (1975) and others indicate the necessity of such corrections. This paper deals with the tests of fine uniform sand at three initial densities, characterized by the relative densities $I_D=0.8$; 0.55 and 0.23, $\gamma_s=26.6$ kN/m^3 $e_{max}=0.89$, $e_{min}=0.56$. The tests have been performed with the experimental apparatus by the Kryzhanovsky-Vorontsov design (Lomize, 1969), which makes possible to apply independently three principal stresses σ_1, σ_2, σ_3 and to define the strains corresponding to them with the 10 cm cubical test specimens. The possibility of the specimen deforming in one of the directions was excluded for achieving the plane strain conditions, and the resulting stress was measured by two hydraulic capsules. The following invariants of the stress-strain behaviour and were used in the study: $\sigma=(\sigma_1+\sigma_2+\sigma_3)/3$ is average stress; $\sigma_i=(1/\sqrt{6})\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$ is shear stresses intensity; $\mu_\sigma=(2\sigma_2-\sigma_1-\sigma_3)/(\sigma_1-\sigma_3)$ (at $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$) is parameter of stress-strain behaviour form; $\varepsilon_v=\varepsilon_1+\varepsilon_2+\varepsilon_3$ is volumetric strain; $\varepsilon_i=(2/\sqrt{3})\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2}$ is shear strain intensity;

Fig.1 shows three stress paths, which characterize the performed three test series, two of them being of the plane strain type. In the test series 1, after the test specimen was subjected to uniform pressure till $\sigma_0=40$; 80 and 160 kPa at $I_D=0.8$ and till $\sigma_0=40$; 80 and 120 kPa at $I_D=0.55$ and $I_D=0.23$ strain at one of the lateral directions was excluded (in this direction the reactive stress σ_2 was recorded), at the same time the other lateral stress σ_3 was maintained constant, and the vertical stress σ_1, was gradually increased up to the failure. In the test series II the uniform pressure was applied till $\sigma_0=160$; 320 and 480 kPa. Then at the stage of loading under the plane strain, the vertical stress σ_1, remained unvariable and σ_2 decreased up to the failure.

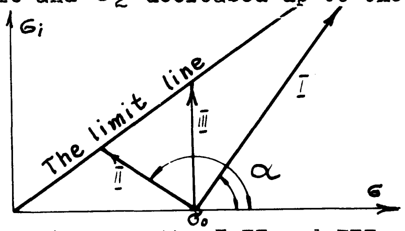

Fig.1. The stress paths I, II and III

The stress path under plane strain in coordinates $\sigma - \sigma_i$ is evident to depend not only on the sequence of variation of the stresses σ_1 and σ_3, but also on the deformation properties of the tested soil, which influence the reactive stress σ_2. The obtained test paths I and II were practically linear and they can be characterized by the angle α between the path and the positive direction of the axis σ. In the purely deviatoric stress tests (path III), the stresses σ =80; 160 and 320 kPa were kept constant at $\mu_\sigma=-1$. For the shear strain description, the relation proposed by Botkin (1939) was used in the form:

$$\sigma_i=\frac{\sigma_i^*}{B+\varepsilon_i}\varepsilon_i \qquad \text{or} \qquad G=\frac{\sigma_i}{\varepsilon_i}=\frac{\sigma_i^*}{B+\varepsilon_i} \qquad (1)$$

were $\sigma_i^*=\sigma \tan\rho$ is the soil strength according to the strength condition of Mises-Schleiher, B is coefficient coinciding in magnitude with the quantity ε_i at $\sigma_i=\sigma_i^*/2$. The volumetric strain ε_v is considered as the sum of the volumetric strain ε_v^0 at the uniform pressure and the additional volumetric strain ε_v^D due to the action of the deviatoric stress. The strain ε_v^0 is adequately approximated by:

$$\varepsilon_v^0=\sigma/(a\sigma+b) \qquad (2)$$

The following relation is proposed for the description of the relationship between ε_v^D and ε_i

$$\varepsilon_v^D=(\psi\varepsilon_i^\beta+\lambda^*\varepsilon_i)\left[1+(\sigma/\sigma')(1+\mu_\sigma)\tan\rho\right] \qquad (3)$$

where ψ, β are experimental parameters; $\lambda^*=\Delta\varepsilon_v^D/\Delta\varepsilon_i$ is dilatancy rate at failure; σ and μ_σ correspond to the current value of ε_i; $\sigma'=1$kPa. The relations (1) and (3) for practical use should be supplemented by the equation of the deviatoric stress path:

$$\sigma = \sigma_0 + \sigma_i \cdot \cot \alpha \qquad (4)$$

All coefficients included in the above relations, are tabulated in Tables I, II. Table III presents the relationship between μ_σ and ε_i, averaged for each test group.

TABLE I

Coefficients in Formula (2) for the paths I, II and III

I_D	0.8	0.55	0.23
a	66,9	37.7	30.4
b, kPa	10300	6420	4440

TABLE II

Coefficients in Formula (3)

Path	I_D	100Ψ	β	$100\lambda^x$	$\tan\rho$	$B\times10^3$	$\cot\alpha$
	0.8	8.5	0.82	18	0.78	3	1.07
I	0.55	4	0.75	9	0.73	4	1.04
	0.23	1.5	0.59	2.4	0.70	6	0.97
	0.8	10.4	0.76	27	0.73	5	-1.11
II	0.55	4.7	0.62	16	0.68	6	-1.11
	0.23	4.0	0.58	4	0.65	8	-1.15
	0.8	3.8	0.49	34	0.87	5	0
III	0.55	2.5	0.50	14	0.80	6	0
	0.23	4.5	0.57	0.4	0.76	9	0

TABLE III

Parameter of the stress-strain behaviour form ($-\mu_\sigma$ x100) at ε_i x100

Path	I_D \ ε_ix100	0.5	1.0	2.0	3.0	5.0
I	0.8;0.55	79	68	58	53	50
	0.23	75	64	52	45	40
II	0.8;0.55;0.23	20	30	30	30	30

The test results enabled to determine the ratio $\sigma_2/(\sigma_1+\sigma_3)$ depending on the strain ε_i. The above ratio has the sense of the Poisson's ratio ν under the plane strain condition:

$$\nu = \sigma_2/(\sigma_1+\sigma_3) \qquad (5)$$

In addition to this, taking the generalized Hook's law as the basis, the Poisson's ratio can be also expressed in terms of stresses σ_1, σ_3 and strains

$$\nu = \frac{\varepsilon_1\sigma_3 - \varepsilon_3\sigma_1}{(\varepsilon_1-\varepsilon_3)(\sigma_1+\sigma_3)} \qquad (6)$$

Relations of ν versus ε_i for the tested dense sand are shown in Fig.2a,b the curves 1 corresponding to relation (6), and the curves 2 to relation (5). The value of ratio $\sigma_2/(\sigma_1+\sigma_3)$ is found to remain constant for the entire test period. But the value ν calculated from the equation (6) is varying during the loading process from the value close to zero and to 0.5 and cannot be called the true

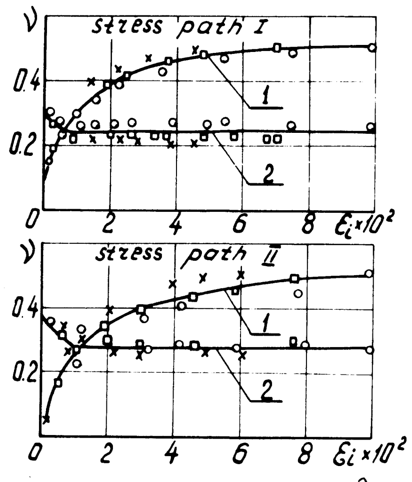

Fig.2. The realtionship between ν and ε_i;
1-relation (6),
2-relation (5).

Poisson's ratio, it is only a parameter of the generalized Hook's law. Hence, the results of the performed investigations have revealed that for the plane strain problem, the deformation characteristics should be determined using the proper test schemes where the stress path, typical for the problem to be solved were taken into consideration.

REFERENCES

Lomize, G.M., Kryzhanovsky A.L., Vorontsov E.I., Goldin A.L.(1969). Study on deformation and strength of soils under three-dimensional state of stress. Proc. 7th ICSMFE, (1), 257-265, Mexico.

Cornforth, D.H.(1964). Some experiments on influence of strain conditions on the strength of sand. Geotechnique, vol.14.

Green, G.E., Reader D.W.(1975), Boundary conditions, anisotropy and sample shape effect on the stress-strain behaviour of sand in triaxial compression and plane strain. Geotechnique, vol.25, No.2.

Botkin, A.I.(1939). Study of stress state in non-cohesive and cohesive soils. Izvestija NIIG, vol.24, pp.153-171.