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A CIRCULAR RIGID PLATE ON A NONLINEARLY DEFORMING BASE

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Stresses and deformations in a nonlinearly elastic half-space due to a rigid loaded plate are computed by the numerical integration of a system of nonlinear differential equations. For the deformations the validity of Botkin's generalized law has been assumed. Calculations cover cohesionless and cohesive soils and soil with both friction and cohesion. Influence on contact pressures, settlements and strains are discussed.

The linear theory of elasticity is widely used for calculating the stressed state and the settlement of structures. In such cases, however, restrictions have to be introduced with the aim of approximating the actual behaviour of the soil subjected to loading. One of these restrictions is the determination of the thickness of the compressible layer of soil, beneath which the soil is conventionally regarded to be incompressible, although its properties may not practically differ from the properties of the soil included in the compressible zone. The magnitude of the load to which the linear theory of elasticity applies in calculating the stress distribution is also limited (Standards, 1962), etc.

Detailed investigations of the stress-strain relationship in soils indicate that it is non-linear (BOTKIN, 1940; VYALOV, 1970; LOMIZE et al., 1970). Therefore the stressed state of soil masses and their deformation can be calculated correctly by assuming nonlinearity for the medium. The solution of such problems is complicated, since it requires the integration of a system of nonlinear differential equations. So far, it has been possible to obtain solutions in the finite form only for some special cases where the medium was assumed to be weightless. The influence of weight in problems concerned with foundation design is very significant and should not be neglected. The fact that the stresses vary within considerable limits should also be taken into account in solving problems for rigid structures. This, for example,

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follows from the diagram of contact pressures under a rigid plate on a linearly deforming base with its infinite ordinates along the edges. Therefore, the nonlinear stress-strain relationship should have an effect on the distribution of the contact pressures and also on the distribution of stresses in the depth.

In this paper, results for problems of a circular rigid centrically loaded plate, resting on the surface of a nonlinearly-deforming half-space with weight are given. It is assumed that no slip, no horizontal displacement occurs along the contact surface.

The basic equations, the description of the method, the solution and some results are given in a previous paper (SHIROKOV et al., 1970).

The radius of the plate is R. The half-space is substituted by a layer of soil, with the thickness $8R$. This gives practically the same results as the half-space. In the horizontal direction, the half-space is substituted by a cylinder with a radius of $16R$. On the external boundary of the zone under consideration, i.e., the cylinder, the boundary conditions $\partial w/\partial r = 0$, $u = 0$ are specified. On the lower base of the cylinder, there are no displacements, i.e. $u = 0$ and $w = 0$. Finally, on the upper free boundary $z = 0$:

$$\left.\begin{aligned} w = w_0; \quad U &= 0 \quad \text{at} \quad r < R \\ \sigma_z = 0; \quad \tau_{rz} &= 0 \quad \text{at} \quad r > 0 \end{aligned}\right\} \tag{1}$$

Here z is the vertical axis with its origin on the surface of the half-space, r is a radius, measured from the axis z in any plane $z =$ const, u is displacement in the direction of r, and w is displacement in the direction of z (Fig. 1).

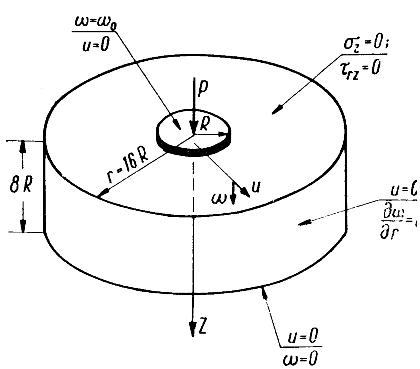

Fig. 1. Coordinate axes r and z and boundary condition

For the numerical calculations we take the following values: $R = 7.5$ m, and the bulk density of the soil $\gamma = 1.66$ tons/m^3. The volumetric forces are directly included in the basic equations (SHIROKOV et al., 1970), similar to LAME's equations. Displacements due to external load were calculated as the difference between the total displacements, obtained in integrating the basic system (with volumetric forces included); the displacements due to the own weight of the soil were calculated by using the same stress-strain relationship.

The aim of these calculations was to establish the influence of the nature of the variation and the magnitude of the modulus of shear deformation G and the modulus of compressibility K as well, on the stressed-strained state of the considered half-space. In all calculations the modulus K was taken as a constant and equal to $K = 300$ kg/cm². The modulus of shear G, as experimentally established by LOMIZE et al. (1970), is a function of all three invariants of the stress or strain tensors. Thus

$$G = G(\sigma_i, \sigma, \mu), \quad G = G(\varepsilon_i, \varepsilon, \mu) \tag{2}$$

where

$$\sigma = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3); \; \sigma_i^2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \tag{3}$$

$$\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3; \; \varepsilon_i^2 = \frac{2}{3}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2] \tag{4}$$

$$\mu = \frac{2\,\sigma_2 - (\sigma_1 + \sigma_3)}{\sigma_1 - \sigma_3} = \frac{2\,\varepsilon_2 - (\varepsilon_1 + \varepsilon_3)}{\varepsilon_1 - \varepsilon_3}$$

For the modulus of shear G, a relationship proposed by FRADIS (1968) was used. He confirmed the validity of this relationship experimentally for sands; it represents the generalization of the assumption suggested by BOTKIN (1940):

$$G = \frac{\sigma_i}{\varepsilon_i} = \frac{A\,\sigma}{B + \varepsilon_i}\left\{1 + m\left[1 + \frac{\mu(9 - \mu^2)}{(3 + \mu^2)^{3/2}}\right]^n\right\}^{-1} \tag{5}$$

where A, B, and n are experimental parameters. At $m = 0$, Eq. (5) becomes identical with BOTKIN's proposal. Parameter values for the investigated sand that enter into Eq. (5) are: $A = 0.96$; $B = 0.0075$; $m = 0.34$; $n = 0.48$.

If the soil is cohesive then, according to BOTKIN, Eq. (5) can be written as

$$G = \frac{\sigma_i}{\varepsilon_i} = \frac{A\,\sigma + A_1\,\sigma_0}{B + \varepsilon_i} \tag{6}$$

where σ_0 is a unit pressure, to ensure the right dimension. It can be taken as 1 kg/cm². A_1 is a dimensionless parameter.

Calculations were carried out for the following relationships $\sigma_i\ (\varepsilon_i)$:

$$\text{(a)}\quad \sigma_i = \frac{\sigma_0\,\varepsilon_i}{0.0075 + \varepsilon_i} \qquad \text{(b)}\quad \sigma_i = \frac{(0.45\,\sigma + \sigma_0)\,\varepsilon_i}{0.0075 + \varepsilon_i}$$

$$\text{(c)}\quad \sigma_i = \frac{(0.96\,\sigma + \sigma_0)\,\varepsilon_i}{0.0075 + \varepsilon_i} \qquad \text{(d)}\quad \sigma_i = \frac{2\,\sigma_0\,\varepsilon_i}{0.0075 + \varepsilon_i} \tag{7}$$

$$\text{(e)}\quad \sigma_i = \frac{0.96\,\sigma\varepsilon_i}{0.0075 + \varepsilon_i} \qquad \text{(f)}\quad \sigma_i = \frac{0.96\,\sigma\varepsilon_i}{0.0075 + \varepsilon_i} f(\mu)$$

In Eq. (7f), function $f(\mu)$ has the same meaning as in Eq. (5) at $m = 0.34$ and $n = 0.48$. Thus, Eq. (7) permits to investigate the influence of the coefficients on the stressed-strained state. The meaning of these coefficients can be determined as follows. If we assume, as Botkin did, that the ultimate state occurs at $\varepsilon i \leqq \infty$, then from Eq. (7) the following relationship is obtained in the general form:

$$\sigma_i = A(\mu)\,\sigma + A_1(\mu)\,\sigma_0 \tag{8}$$

where $A(\mu)$ and $A_1(\mu)$ are experimentally established strength parameters that are governed by the type of the stressed state.

If Malyshev's strength criterion is used (Malyshev, Fradis, 1968), which was derived on the basis of different concepts, we get:

$$\sigma_i = \frac{\sqrt{3(3+\mu^2)}\cdot\dfrac{\sin\varrho}{X}}{3 + \mu\dfrac{\sin\varrho}{X} - 3\,\dfrac{(1-X)(1-\mu^2)}{4}}\,\sigma + \\ + \sigma_0\bar{C}\,\frac{\sqrt{3(3+\mu^2)(X^2-\sin^2\varrho)}}{X\left[3 + \mu\dfrac{\sin\varrho}{X} - 3\,\dfrac{(1-X)(1-\mu^2)}{4}\right]} \tag{9}$$

Thus, the values of A and A_1 can be found by means of the invariant strength values ϱ, x and C introduced by Malyshev. If Eqs. (8) and (7) are compared, it can be said that Eq. (7) a and d refer to materials having cohesion, e and f to materials having friction, and b and c both friction and cohesion. It should be noted that in Eq. (7f), the type of stressed state is taken into account, while in the other equations it is assumed that the parameters are independent of μ.

The results of calculations carried out by a computer are given in Figs. 2–5 and are as follows:

1. Fig. 2 shows settlement curves due to the average pressure under the plate and complying with the deformation (Eqs 7a through f). From them it follows that the relationship between the settlement and the load is

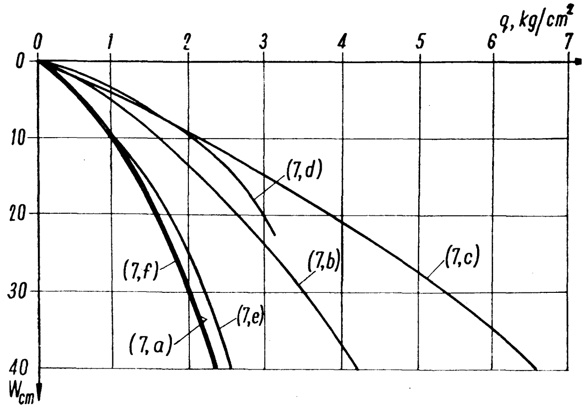

Fig. 2. Settlement w vs. unit load q for deformation laws according to Eqs. (7a), (7b), (7c), (7d), (7e) and (7f)

curvilinear and an increase in friction (7a, b and c) sharply reduces the magnitude of settlement at the same load.

2. The comparison of curves 7e and f reveals that in the case of axial symmetrical deformation the type of stressed state has a small influence on the change in settlement. According to the analysis, carried out by SHIROKOV, the parameter μ was close to $\mu = -1$, over almost the whole range.

3. Diagrams of contact pressures are given in Fig. 3 for three deformation laws: soils, having friction only (Fig. 3a), Eq. (7e); soils having cohesion only (Fig. 3b) Eq. (7a); and soils with both friction and cohesion, (Fig. 3c) Eq. (7c). Comparison of these diagrams shows that in the first case the distribution of contact pressures tends to a parabolic form with an increase in load. The diagrams for the other two relationships, i.e., for cohesive soil, show the same feature; however, the saddle shape is much more accentuated. Upon an increase in load the contact pressures under the plate are more evenly distributed.

4. Displacements in the axis of the plate (Fig. 4a) have qualitatively the same character as in a linear medium, however, the vertical displacements w with depth decrease much more rapidly than in a linearly deforming medium. It should be noted that the settlement diagrams shown in Fig. 4a refer to a case where the total settlement is equal to 12 cm. In Fig. 3 this corresponds to various values in load (which are given in the caption to

Fig. 4). Compression of a soil layer, having a thickness $3R$ produces 94 per cent of the plate settlement, and only 6 per cent of the settlement is due to the compression of the underlying soil. For a linearly deforming medium

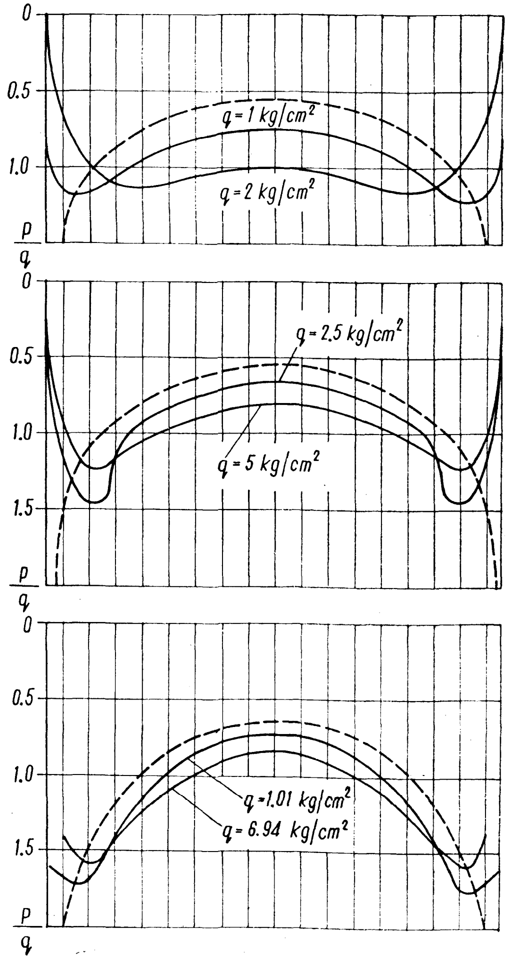

Fig. 3. Diagrams of contact pressures p under the plate referred to the average pressure q for the following deformation laws: a — deformation law Eq. (7e); b — (7d); c — (7c). Dotted line was obtained for the linear theory of elasticity

these values comprise 70 per cent and 30 per cent, respectively. Calculations carried out both for a loose soil and for a cohesive soil confirm the existence of the so-called "active" zone. On this experimentally proved fact, the model of a soil base in the form of a layer of soil of finite thickness is based. According to this model, the deformation of the subsoil beneath a certain depth can be practically neglected.

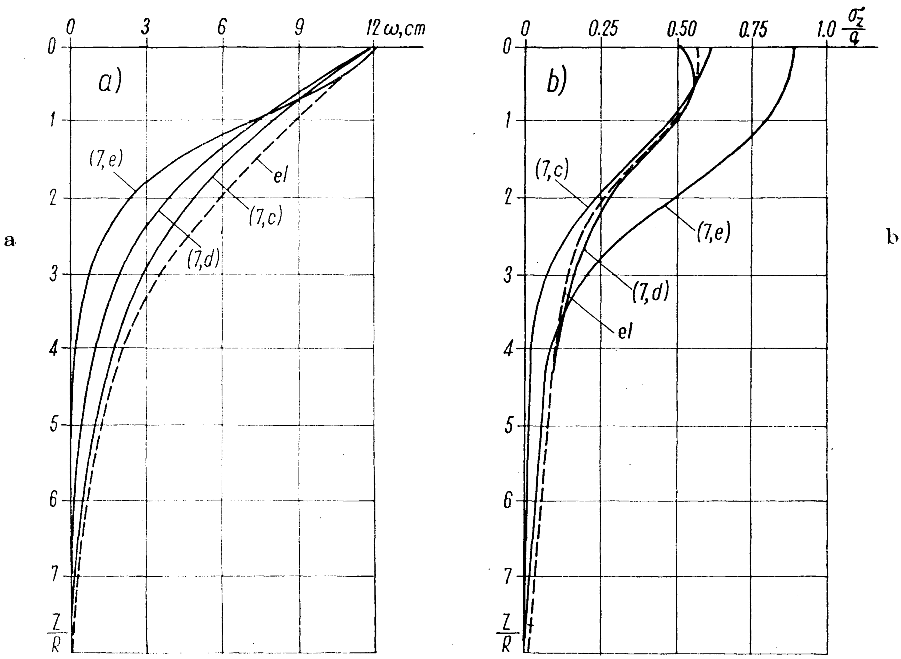

Fig. 4. Distribution of displacements (a) and stresses (b) with depth z/R in the axis of the plate for the following loads: $q = 1.24$ kg/cm² (Eq. 7e); $q = 2.35$ kg/cm² (Eq. 7d); $q = 2.46$ kg/cm² (Eq. 7c). Dotted line complies with the linear theory of elasticity

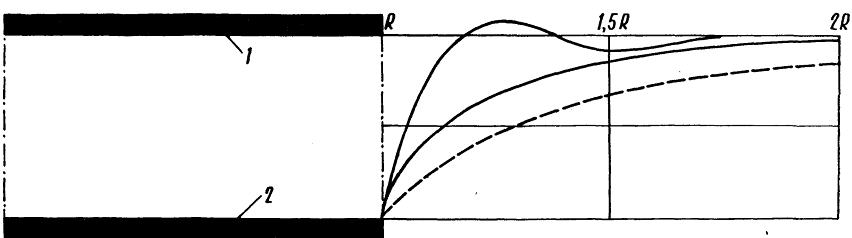

Fig. 5. Deformation of the surface of the half-space around the plate for $q = 1.24$ kg per cm² (Eq. 7e) and $q = 2.35$ kg/cm² (Eq. 7d). Dotted line complies with the linear theory of elasticity. 1. Position of plate before loading. 2. Position of plate after a settlement of 12 cm

5. The displacement of the soil surface around the edge of the plate (Fig. 5) decreases quickly. For loose soil, the formation of a characteristic "ridge" has been established.

Thus, the disagreement between the results of experiments and calculations, based on the hypothesis of a linearly deformable medium can be attributed to a large degree to the nonlinearity of the shear deformation in soils.

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