Plastic and Elastic-Plastic Problems in the design of Foundation

Problèmes plastiques et elastico-plastiques dans le calcul des fondations

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Summary

The authors give a simple method of designing foundations, taking account of normal and tangential loads, as well as bending moments.

They consider that elastic-plastic problems may be solved by employing a blunt wedge uniformly loaded at an incline to one of the sides. On the basis of this solution, it was possible to determine the magnitudes and limits of the elastic and plastic zones for any load, as well as the stresses in these zones. The authors assume that the soil posses both friction and cohesion.

Sommaire

Dans la première partie du rapport due à M. Malichev, on trouve l'exposé d'une méthode simple de calcul du pouvoir portant du sol de fondation sollicité par des forces tangentielles et normales ainsi que par des moments.

Dans la seconde partie du rapport, I. Fédorov expose la solution de problèmes élastico-plastiques pour un coin émoussé non pesant, soumis à une charge inclinée uniforme suivant une de ses arêtes extérieures. Se basant sur cette solution il devient possible de déterminer les limites et les dimensions des zones élastique et plastique pour une charge quelconque sur l'arête extérieure de même que l'état de contrainte dans les zones mentionnées.

Dans les deux parties il est admis que le sol possède un frottement et une cohésion.

I

The problem of calculating the ultimate bearing capacity of a foundation, solved by V.V. Sokolovsky, assumes an inclined load for a soil foundation with cohesion C, as well as side surcharge on the foundation. V. V. SOKOLOVSKY (1954) advises an approximate method for calculating the bearing capacity of the foundation supported by soil with a coefficient of friction φ and cohesion, based on the possibility of determining the bearing capacity of an ideal granular soil ($c = 0$, $\gamma \neq 0$, $\varphi \neq 0$) and of a cohesive soil ($c \neq 0$, $\gamma = 0$, $\varphi \neq 0$), γ – volume weight of foundation soil.

In order to calculate the bearing capacity of the foundation Q_f, under an eccentric load Q_a embedded in the soil at depths D_1 and D_2 on the left and right sides respectively (Fig. 1a), a simple graphical method can be employed. The limiting value of the vertical ordinates of the diagram of reaction $q_{f\cdot z}$ is expressed by the following formula (Fig. 1, b) :

$$q_{f,z} = N'_q \gamma D_1 + N'_c c + N'_\gamma \gamma x \quad \dots . \quad (1)$$

Coefficients N'_q, N'_c, N'_γ are given in Table 1 and are derived from a book by V. V. SOKOLOVSKY (1954). They vary with the inclination of the total resultant pressure, the vertical component Q_{fz} acting at an angle δ.

V. V. Sokolovsky usually considers the case of a semi-infinite load, corresponding to a foundation of infinite width. In practive, foundations have a finite width B which should be borne in mind by the designer. If the load on the foundation is vertical and symmetrical and the surcharge at both sides is the same, then on the basis of tests, either one-sided or two-sided displacement may occur. In the former case, the bearing capacity remains practically the same. The case of a two-sided displacement has been considered both by Prandtl and by R. Hill.

In experiments (MALYSHEV, 1953) performed in the VODGEO INSTITUTE during 1948 and 1949, showing the presence of an

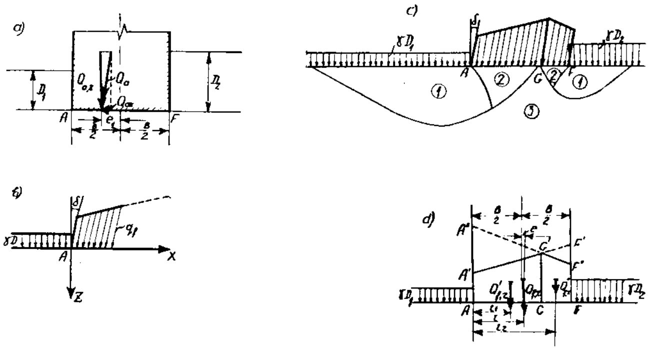

Fig. 1 (a) loads on the foundation ;
(b) scheme for calculating ultimate bearing capacity according to V. V. Sokolovsky ;
(c) zones in footing : 1) equilibrium zone ; 2) core under foundation ; 3) elastic zone ;
(d) scheme for finding ultimate bearing capacity of a foundation footing.
(a) Fondation et sollicitations ;
(b) schéma pour la détermination de la charge critique d'après V. Sokolovski ;
(c) zones des fondations : 1) zone de l'état limite ; 2) noyau sous les fondations ; 3) zone élastique ;
(d) schéma pour la détermination du pouvoir portant de la fondation.

elastic core under a stamp, tests proved that the above method could be applied.

The following scheme also does not exclude the possibility of an elastic core under the stamp. Fig. 1, c shows the zones with limiting condition — 1, with infinite condition — 3 and zone 2, which theoretically has a minimum limiting condition

Table 1

φ°	δ°	N'_q	N'_c	N'_γ	N''_q	N''_c	N''_γ
0	0	1·00	5·14	0	1·0	5·14	0
10	0	2·47	8·34	0·46	2·47	8·34	0·45
	10	1·50	2·84	0·17	1·65	3·69	0·61
20	0	6·40	14·8	2·94	6·40	14·8	2·94
	10	4·65	10·0	1·32	7·79	18·7	5·43
	20	2·09	3·0	0·32	3·05	5·64	5·59
30	0	18·4	30·1	16·2	18·4	30·1	16·2
	10	12·9	20·6	6·91	23·9	39·7	27·3
	20	7·97	12·1	2·72	28·3	47·3	40·0
	30	2·75	3·02	0·43	6·70	9·85	35·8
40	0	64·2	75·3	76·4	64·2	75·3	76·4
	10	42·4	49·3	37·3	90·5	105	136
	20	25·4	29·1	15·2	117	139	160
	30	13·1	24·4	4·28	141	167	251
	40	3·42	2·88	0·49	18·8	21·2	195

of stress, but which is generally included in the core. However, the shape of the core differs somewhat from the shape in the experiments where the core was solid, and therefore, it may be assumed that the calculated bearing capacity will be slightly lower than the actual value. The authors employed this scheme for finding the bearing capacity of the foundation, loaded at an incline, asymmetrically and with different loads at the sides, due to its extreme simplicity.

It should be noted that for this scheme (Fig. 1, c) it is assumed that displacement is possible at both sides of the foundations.

Due to the assumption of two-sided displacement (Fig. 1, d) calculations by formula (1) are inadequate, as it is necessary to know the ordinates corresponding with the right side of the limit diagram. The following formula is used :

$$q_f = N_q''\gamma D_2 + N''_c C + N''_\gamma \gamma(B - x) \quad .... \quad (2)$$

Thus, for plotting the diagram of normal stresses, substitute in formula (1) $x = 0$, $x = B$ and correspondingly assume $q'_{f,z} = AA'$ and $q''_{f,z} = FF'$ (Fig. 1, d), while in formula (2), $x = B$, $x = 0$, obtaining $q'''_{f,z} = FF''$ and $q^{IV}_{f,z} = AA''$. The values of coefficients N''_q, N''_c and N''_γ are also given in Table 1. They have been calculated by the formulae derived by SOKOLOVSKY (1954). The tangential stresses $q_{f,x}$ at all points below the foundations are expressed by the following :

$$q_{fx} = q_{fz} \, tg\delta \quad .... \quad (3)$$

Having found $q'_{f,z} q''_{f,z} q'''_{f,z} q^{IV}_{f,z}$ and having plotted them graphically in the required scale (Fig. 1, d), the points of intersection of the lines are determined. Further, assuming $x = AG$, it is possible by formula (1) to determine $q^V_{f,z} = GG'$. Then the value of the resultant reaction can be found, comprising the total sum of the areas of the left and right trapeziums and the point of application $Q_{f,z} = Q'_{f,z} + Q''_{f,z}$.

The abscissas of the centres of gravity of the areas of these trapeziums, i.e. the points of application of loads $Q'_{f,z}$ and $Q''_{f,z}$ are found graphically. The coordinates of the points of application of the total resultant $Q_{f,z}$ are determined from the ratio :

$$l = e + \frac{B}{2} = \frac{Q'_{f,z} l_1 + Q''_{f,z} l_2}{Q'_{f,z} + Q''_{f,z}} \quad .... \quad (4)$$

where e — eccentricity in relation to the middle of the foundation footing

$$Q'_{f,z} = \frac{1}{2}(AA' + GG') \times AG;$$

$$Q''_{f,z} = \frac{1}{2}(GG'' + FF'')GF;$$

The diagrams must then be compared with the actual values, which are determined by force $Q_{a,z}$ and bending moment M, the former applied to the foundation, and the latter to the centre of the foundation footing. It is clear that for the general case of actual eccentricity (Fig. 1, a)

$$e_1 = \frac{M}{Q_{a,z}}$$

is not equal to that obtained for the limiting condition, i.e. $e_1 \neq e$.

For estimating the influence of eccentricity, the authors used the results of the experimental investigations by A.A. NICHIPOROVITCH and N. J. KHRUSTALJOV (1957). By using this information, it was possible to plot a convenient graph (Fig. 2, a) for finding coefficient $\alpha \leqslant 1$, where :

$$\alpha = \bar{Q}_{f,z}/Q_{f,z} \quad .... \quad (5)$$

where $Q_{f,z}$ — value of bearing capacity, and $\bar{Q}_{f,z}$ — value bearing capacity not for an eccentricity of e, but for the given eccentricity of e_1. Coefficient α is determined by this graph as follows : Plot along the abscissa ration $\frac{e}{B}$, then find the nearest curve at the right side intersecting the ordinate axis, equal to unity for an abscissa equal to ratio $\frac{e}{B}$. Along this curve, plot the value of the ordinates corresponding with the

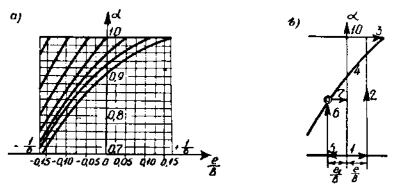

Fig. 2 (a) Curves for finding coefficient α ;
(b) explanation for employing graph, numbers showing order of performed operations.
(a) Graphique pour la détermination du coefficient α ;
(b) clé du graphique, les numéros indiquent l'ordre des opérations exécutées.

given ratio $\frac{e_1}{B}$, which equals α. The explanations for the given curve are in Fig. 2, b. The unknown bearing capacity $\bar{Q}_{f,z}$ is calculated by formula :

$$\bar{Q}_{f,z} = \alpha Q_{f,z} \quad .... \quad (6)$$

If it is found that $e_1 > e$, then in this case it is necessary to employ a value of unity for α.

The factor of safety K_s is determined by the expression :

$$K_s = \frac{\bar{Q}_{f,z}}{Q_{a,z}}$$

where $Q_{a,z}$ is the vertical load on the foundation.

II

When considering the theoretical basis of the elastic-plastic equilibrium of a blunt wedge, it is necessary to estimate the elastic of plastic condition from some function f, in accordance with Coulomb's theory given by

$$f = \sqrt{\frac{1}{4}(\sigma_r + \sigma_\theta)^2 + \tau^2_{r\theta}} - \sin\varphi\left(\frac{\sigma_r + \sigma_\theta}{2} + c \cdot \mathrm{ctg}\,\varphi\right) \qquad \ldots\ldots (7)$$

For a two-dimensional as well as a three-dimensional stressed condition, based on the work of I.V. Fjedorov (1957) the value of the function will differ from the value in (7). If in the reduced function f, the difference in the right part is greater than zero, the soil possesses some residual shear strength and therefore has some elasticity.

The criterion for estimating the plastic condition of the soil mass is $f = 0$. In order to describe the mechanical properties of the soil which are both frictional and cohesive, it is necessary to employ a model of an ideal elastic-plastic body.

The behaviour of thas ideal model differ slightly from what occurs in practice, as the stress-strain diagram does not consist of two straight sections, since tests have revealed that there is no abrupt break in the line where the material passes into the plastic condition.

When solving the problem, of a blund wedge polar co-ordinates r and θ have their origin at 0 (Fig. 3). Stress distribution in the blunt with an incline of α loaded on the surface with an oblique uniform load, supported by an elastic medium, may be determined by the following formulae based on the elastic theory :

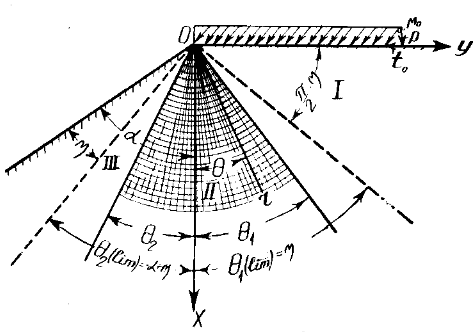

Fig. 3 Method of calculating stress for a blunt wedge.
Schéma de calcul pour un coin émoussé sollicité sur son arête extérieure par une charge inclinée.

$$\left.\begin{matrix}\sigma_r \\ \sigma_\theta\end{matrix}\right\} = p + t_0\left(\pi + \frac{1}{\sin 2\alpha}\right) + \frac{-p - t_0\Phi}{\psi}[\pi + \mathrm{ctg}\,\alpha - 2\theta \pm (\sin 2\theta - \mathrm{ctg}\,\alpha\cos 2\theta)] - t_0\left(2\theta \pm \frac{\cos 2\theta}{\sin 2\alpha}\right)$$

$$\tau_{r\theta} = t_0\left(1 + \frac{\sin 2\theta}{\sin 2\alpha}\right) + \frac{-p - t_0\Phi}{\psi}(1 + \cos 2\theta + \mathrm{ctg}\,\alpha\sin 2\theta) \qquad \ldots\ldots (8)$$

where :

$$\Phi = 2\alpha + \pi + \mathrm{ctg}\,\alpha$$

$$\psi = 2\alpha + \pi + 2\mathrm{ctg}\alpha$$

The plastic condition is determined as the maximum of function f. In order to find this maximum value, substitute values σ_r, σ_θ and $\tau_{r\theta}$ from (8) and (7) and deriving $\frac{df}{d\theta}$ and solving the equation in relation to angle θ, the plastic condition in the soil appears at an angle of θ_1 given by the expression

$$\theta_1 = F(\mathrm{tg}\,\mu_0, \alpha, \sin\varphi) \qquad \ldots\ldots (9)$$

where F is some function, determined by the expression of the stress component tan

$$\mu_0 = \frac{t_0}{P}.$$

As load P increases, the plastic zone increases and reaches its limit at the right of the vertical axis to angle θ_1, and to the left to angle θ_2 as shown in Fig. 3.

The stresses in the right and left elastic fields (Field 1 when $\theta_1 \leqslant \theta \leqslant \frac{\pi}{2}$ and field III when $\theta_2 \geqslant \theta \geqslant -\alpha$) may be determined using the formulae of the elastic theory, as follows :

$$\left.\begin{matrix}\sigma_r \\ \sigma_\theta\end{matrix}\right\} = A_1 - 2A_2\theta \pm (A_3\sin 2\theta + A_4\cos 2\theta)$$

$$\tau_{r\theta} = A_2 + A_3\cos 2\theta - A_4\sin 2\theta \qquad \ldots\ldots (10)$$

In plastic field II $\theta_2 \leqslant \theta \leqslant \theta_1$ the stresses may be found by means of equilibrium equations and the equilibrium limit. The components of stresses σ_r, σ_θ, $\tau_{r\theta}$ can be calculated from the formulae due to V. V. Sokolovsky (1954) as follows :

$$\left.\begin{matrix}\sigma_r \\ \sigma_\theta\end{matrix}\right\} = \sigma[1 \pm \sin\varphi\cos 2\psi] - c \cdot \mathrm{ctg}\,\varphi\,;$$

$$\tau_{r\theta} = \sigma\sin\varphi\sin 2\psi \qquad \ldots\ldots (11)$$

where :

ψ incline of highest main normal stress σ_1 to radius r.

Fig. 4, $\mu = \frac{\pi}{4} - \frac{\varphi}{2}$ is the incline angle of main stress σ_1 to the first set of sliding lines.

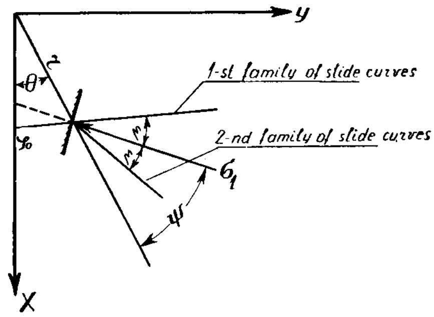

Fig. 4 Conditions of stress at a point in plane $x\ y$.
Etat de contrainte en un point sur la surface $x\ y$.

Consider that $\sigma_r = \sigma_r(\theta)$; $\sigma_\theta = \sigma_\theta(\theta)$ and $\tau_{r\theta} = \tau_{r\theta}(\theta)$;
From the equilibrium equations, taking into account functions (11), we obtain :

$$\frac{d\sigma}{d\theta}\sin\varphi\sin 2\psi + 2\sigma\sin\varphi\cos 2\psi\left(\frac{d\psi}{d\theta} + 1\right) = 0$$

$$\frac{d\sigma}{d\theta}(1 - \sin\varphi\cos 2\psi) + 2\sigma\sin\varphi\sin 2\psi\left(\frac{d\psi}{d\theta} + 1\right) = 0 \qquad \ldots\ldots (12)$$

The solution is determined by the physical limitations of the proposed problem, as well as by the boundary conditions. The following solution is suitable, where :

$$\psi = \frac{\pi}{4} + \frac{\varphi}{2} \quad \ldots. \quad (13)$$

Substituting value ψ in equation (12), we find that :

$$\sigma = M_1 e^{2\theta \operatorname{tg} \varphi} \quad \ldots. \quad (14)$$

Thus, the stress components in the plastic field will be as ollows in accordance with (11) and (14) :

$$\left.\begin{matrix}\sigma_r \\ \sigma_\theta\end{matrix}\right\} = M_1 e^{2\theta \operatorname{tg}\varphi} [1 \pm \sin^2 \varphi] - c \cdot \operatorname{ctg}\varphi ;$$

$$\tau_{r\theta} = - M_1 e^{2\theta \operatorname{tg}\varphi} \sin\varphi \cos\varphi ; \quad \ldots. \quad (15)$$

Determination of the stress components in the first and fourth fields by means of equation (10) involves the use of eleven arbitrary constants, four each for the first and third fields (A_1, A_2, A_3, A_4, A'_1, A'_2, A'_3, A'_4), M_1 being the same for the first and third fields. Besides, angles θ_1 and θ_2 must be determined, which set the boundary of the plastic field, determined at $\theta = \pi/_2$; $\sigma_\theta = P$; $\tau_{r\theta} = 0$; and at $\theta = -\alpha$; $\sigma_\theta = 0$; $\tau_{r\theta} = 0$; as well as on condition of continuous stress components at the boundary between the elastic and plastic fields when $\theta = \theta_1$ and $\theta = \theta_2$.

After having found A_1, A_2, A_3, A_4 and M_1, the stress components in field I expressed by θ_1 are as follows :

$$\left.\begin{matrix}\sigma_r \\ \sigma_\theta\end{matrix}\right\} = p + t_0[\pi - 2\theta - (\pi - 2\theta \pm 1)\frac{\sin 2\theta_1}{1 + \cos 2\theta_1} \pm \sin 2(\theta_1 - \theta)$$

$$- M_1 e^{2\theta_1 \operatorname{tg}\varphi} \sin\varphi \frac{\sin(2\theta_1 + \varphi) + \sin\varphi + (\pi - 2\theta)\cos(2\theta_1 + \varphi) \pm [\sin\varphi\cos 2\theta - \sin(2\theta_1 + \varphi - 2\theta)]}{1 + \cos 2\theta_1} \quad \ldots. \quad (16)$$

$$\tau_{r\theta} = t_0\left(1 - \frac{\cos 2(\theta_1 - \theta) + \cos 2\theta_1}{1 + \cos 2\theta_1}\right) - M_1 e^{2\theta_1 \operatorname{tg}\varphi} \sin\varphi \frac{\cos(2\theta_1 + \varphi - 2\theta) + \cos(2\theta_1 + \varphi) + \sin\varphi \sin 2\theta}{1 + \cos 2\theta_1}$$

where :

$$M_1 = \frac{(P + c \cdot \operatorname{ctg}\varphi)(1 + \cos 2\theta_1) - t_0(\pi - 2\theta_1 - \sin 2\theta_1)}{e^{2\theta_1 \operatorname{tg}\varphi} \{1 + \cos 2\theta_1 + \sin\varphi[(\pi - 2\theta_1)\cos(2\theta_1 + \varphi) + \sin(2\theta_1 + \varphi) + \sin\varphi]\}} \quad \ldots. \quad (17)$$

A similar method is used for finding the components of the third elastic field, given by the following :

$$\left.\begin{matrix}\sigma_r \\ \sigma_\theta\end{matrix}\right\} = M_2 e^{2\theta \operatorname{tg}\varphi} \frac{\sin\varphi}{1 - \cos 2(\theta_2 + \alpha)} \{- 2(\alpha + \theta)\cos(2\theta_2 + 2\alpha + \varphi) + \sin(2\theta_2 + 2\alpha + \varphi)$$

$$- \sin\varphi \mp \sin(2\theta - 2\theta_2 - \varphi) \mp \sin\varphi\cos(2\alpha + 2\theta)\} \quad \ldots. \quad (18)$$

$$\tau_{r\theta} = - M_2 e^{2\theta_2 \operatorname{tg}\varphi} \frac{\sin\varphi}{1 - \cos 2(\theta_2 + \alpha)} \{\cos(2\theta_2 + \varphi - 2\theta) - \sin\varphi\sin(2\alpha + 2\theta) - \cos(2\theta_2 + \varphi + 2\alpha)\}$$

where :

$$M_2 = \frac{c \cdot \operatorname{ctg}\varphi[1 - \cos 2(\theta_2 + \alpha)]e^{-2\theta_2 \operatorname{tg}\varphi}}{1 - \cos 2(\theta_2 + \alpha) + \sin\varphi[2(\alpha + \theta_2)\cos(2\theta_2 + 2\alpha + \varphi) - \sin(2\theta_2 + 2\alpha + \varphi) + \sin\varphi]} \quad \ldots. \quad (19)$$

For plastic field II, the stress components after finding M_1 are :

$$\left.\begin{matrix}\sigma_r \\ \sigma_\theta\end{matrix}\right\} = \frac{\{(P + c \cdot \operatorname{ctg}\varphi)(1 + \cos 2\theta_1) - t_0[(\pi - 2\theta_1) + \sin 2\theta_1]\} e^{2\theta \operatorname{tg}\varphi}(1 \pm \sin^2\varphi)}{e^{2\theta_1 \operatorname{tg}\varphi}\{1 + \cos 2\theta_1 + \sin\varphi[(\pi - 2\theta_1)\cos(2\theta_1 + \varphi) + \sin(2\theta_1 + \varphi) + \sin\varphi]\}} - c \cdot \operatorname{ctg}\varphi$$

$$\tau_{r\theta} = - \frac{\{(P + c \cdot \operatorname{ctg}\varphi)(1 + \cos 2\theta_1) - t_0[(\pi - 2\theta_1) + \sin 2\theta_1]\} e^{2\theta \operatorname{tg}\varphi} \sin\varphi\cos\varphi}{e^{2\theta_1 \operatorname{tg}\varphi}\{1 + \cos 2\theta_1 + \sin\varphi[(\pi - 2\theta_1)\cos(2\theta_1 + \varphi) + \sin(2\theta_1 + \varphi) + \sin\varphi]\}}. \quad \ldots. \quad (20)$$

For finding $\frac{P}{c} = f(\theta_1,\theta_2)$ employ coinciding values M_1 and M_2 determined from the right and left boundary conditions. On this basis it is possible to find the following expression for $\frac{P}{c}$:

$$\frac{P}{c} = - \frac{t_0}{c}\frac{\pi - 2\theta_1 - \sin 2\theta_1}{1 + \cos 2\theta_1} - \operatorname{ctg}\varphi \{ 1 -$$

$$\frac{e^{2\theta_1 \operatorname{tg}\varphi}[1 - \cos 2(\theta_2 + \alpha)]\{1 + \cos 2\theta_1 + \sin\varphi[(\pi - 2\theta_1)\cos(2\theta_1 + \varphi) + \sin(2\theta_1 + \varphi) + \sin\varphi]\}}{e^{2\theta_2 \operatorname{tg}\varphi}(1 + \cos 2\theta_1)\{1 - \cos 2(\theta_2 + \alpha) + \sin\varphi[2(\alpha + \theta_2)\cos(2\theta_2 + 2\alpha + \varphi) - \sin(2\theta_2 + 2\alpha + \varphi) + \sin\varphi]\}} \quad \ldots. \quad (21)$$

From equations (16), (18), and (20) it is possible to obtain stress components $\varphi = 0$ and $t_0 \neq 0$, as well as for $\varphi \neq 0$, $t_0 = 0$ and $\alpha = \frac{\pi}{2}$.

Primarily, the elastic-plastic solution for a blunt wedge loaded at the sides by a uniform load of P and q was given by I. V. FJEDOROV (1958).

The value of $\frac{P}{c}$ fort $t_0 = 0$ and $\varphi \neq 0$:

$$\frac{P}{c}+\operatorname{ctg}\varphi=\frac{e^{2(\theta_1-\theta_2)\operatorname{tg}\varphi}[1-\cos 2(\theta_2+\alpha)]\{1+\cos 2\theta_1+\sin\varphi[(\pi-2\theta_1)\cos(2\theta_1+\varphi)+\sin(2\theta_1+\varphi)+\sin\varphi]\}}{(1+\cos 2\theta_1)\{1-\cos 2(\theta_2+\alpha)+\sin\varphi[2(\alpha+\theta_2)\cos(2\theta_2+2\alpha+\varphi)-\sin(2\theta_2+2\alpha+\varphi)+\sin\varphi]\}}\times\left(\frac{q}{c}+\operatorname{ctg}\varphi\right). \quad \ldots\ldots (22)$$

All stress components are expressed by unknowns θ_1 and θ_2, which are determined by one more equation. From the linear function between θ_1 and θ_2 under the limiting conditions an equation is found linking together θ_1 and θ_2 for a blunt wedge :

$$\theta_2=\frac{\mu-\alpha-\theta_0}{\mu-\theta_0}\theta_1+\frac{\alpha\theta_0}{\mu-\theta_0} \quad \ldots\ldots (23)$$

Taking $\alpha=\frac{\pi}{2}$ and $\theta_0 = -\varphi$, from (23) the following equation of θ_2 for a semi-plane is obtained :

$$\theta_2=-\frac{\mu}{\mu+\varphi}\theta_1-\frac{\pi/2\cdot\varphi}{\mu+\varphi} \quad \ldots\ldots (24)$$

On the basis of this solution for the blunt wedge, a specific case when $\varphi = 0$ is the solution obtained previously by FREUDENTAL (1937), az well as by V. V. SOKOLOVSKY (1954), for a semi-plane, and for a blunt wedge by G. S. SHAPIRO (1952). While $\frac{P}{C}=f(\theta_1,\theta_2)$ when $\alpha=\frac{\pi}{2}$ from (22) is :

$$\frac{P}{c}+\operatorname{ctg}\varphi=\frac{e^{2(\theta_1-\theta_2)\operatorname{tg}\varphi}(1+\cos 2\theta_2)\{1+\cos 2\theta_1+\sin\varphi[(\pi-2\theta_1)\cos(2\theta_1+\varphi)+\sin(2\theta_1+\varphi)+\sin\varphi]\}}{(1+\cos 2\theta_1)\{1+\cos 2\theta_2-\sin\varphi[(\pi+2\theta_2)\cos(2\theta_2+\varphi)-\sin(2\theta_2+\varphi)-\sin\varphi]\}}\times\left(\frac{q}{c}+\operatorname{ctg}\varphi\right). \quad \ldots\ldots (25)$$

For finding the values of the plastic field at the slope depending on the vertical load and soil characteristics formulae (22) and (23) should be employed, and for the semi-plane — (25) and (24). During calculations value θ_1 should be given, and then, using either (23) or (24) with constants φ and c, find θ_2 and $\frac{P}{c}$. Fig. 5 shows the functions of $\frac{P}{c}=f(\theta_1,\theta_2)$ for a blunt wedge with $\alpha=30°$, while in Fig. 6 are shown similar curves for the semi-plane in case φ from 0° to 30° ($q = 0$).

For the same value of $\frac{P}{c}$, the field of plastic deformation widens with, the lower surcharge q, friction angle φ and cohesion c.

The condition of plasticity established by Moore-Rankine is often used for defining the boundary conditions of plasticity. In this case, the stress components are taken from elastic solution and substituted in the Moore-Rankine plastic condition, forming a limiting curve equation of $F_1(r, \theta, \varphi) = 0$ for the selected system of co-ordinates, in the range of which the soil is in plastic condition (F_1 — function determined by the kind of stress component). Solving the equations in relation to r or θ, it is possible to set a limit to the plastic condition.

The determination of the limits of plasticity for the semi-plane on the basis of the above solutions ($\varphi = 30°$, $c = 0{\cdot}2$ kg per sq cm) and load $P = 1{\cdot}5$ kg per sq cm, shows that the value of the plasticity boundary determined from the Moore-Rankine plastic condition with the aid of the equations of stress components of the elastic solution results in fields twice as large as by the proposed solution (Fig. 7). This may be explained by redistribution of stress, a factor which was not taken into account when using the elastic solution.

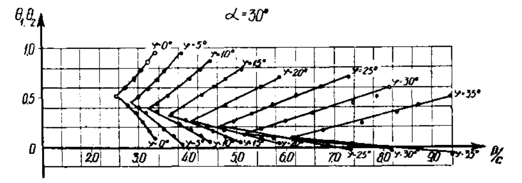

Fig. 5 Curves giving dimension of plastic zones θ_1 and θ_2 depending on $\frac{P}{c}$ for $\alpha = 30°$ and different values of φ.
Graphique de l'interdépendance entre les cotes θ_1 et θ_2 de la zone plastique et $\frac{P}{c}$ pour $\alpha = 30°$ et différentes valeurs de φ.

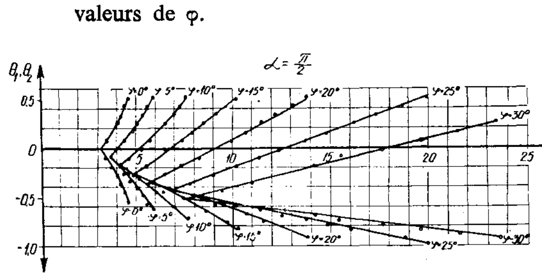

Fig. 6 Curves giving dimensions of plastic zones θ_1 and θ_2 depending on $\frac{P}{c}$ for a semi-plane and different values of φ.
Graphique de l'interdépendance entre les cotes θ_1 et θ_2 de la zone plastique et $\frac{P}{c}$ pour un demi-plan et différentes valeurs de φ.

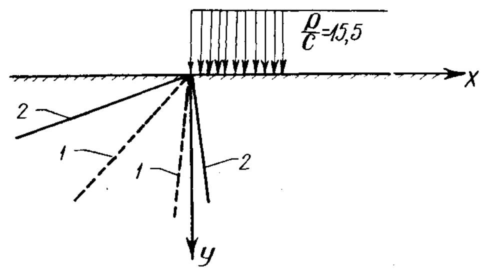

Fig. 7 Comparative curves of plastic zones for elastic-plastic (1) and elastic (2) solutions.
Graphique comparatif des zones plastiques dans les cas de solutions élastico-plastiques (1) et élastiques (2).

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