***Determination of the Angle of Dilatancy by Triaxial Compression Tests***

**Mirnyy A.Y.**, Lomonosov Moscow State University, Moscow, Russia
**Idrisov I.Kh.**, “GEOTEK” OOO, Penza, Russia

All dispersed media, including soils, have a unique mechanical property – dilatancy. This property manifests itself as a volume expansion during shear strain and is explained by the special features of the mutual displacement of solid particles. If there is a decrease in volume, the term "contraction" is used.

It is well known from experimental data that the change in the “sign” of the dilatancy depends on how close the dispersed medium is to the critical density $\rho_{crit}$ (or the critical porosity $e_{crit}$), that defines the medium with formed sliding surfaces and shears. If initially the medium was underconsolidated, contraction may be observed. In the case of initially dense state, dilatancy may occur.

As a mechanical phenomenon, dilatancy was described in 1885 by O. Reynolds. However, one of the first researchers of this subject was D.W. Taylor, who published a series of articles in the 1930s, the totals of which were summarized in the 1948 soil mechanics textbook. But the most well-known publication in the field of dilatancy research is the work by P.W. Rowe, published in 1962 and containing a general theory of dilatancy. According to Rowe's theory, the relationship between volumetric and shear strains for a dilating medium is expressed as follows:

$$\Delta\varepsilon_v = \sin\psi \cdot \Delta\gamma\,,$$

where $\Delta\varepsilon_v$ is the volumetric strain of the soil, $\psi$ is the angle of dilatancy, $\Delta\gamma$ is the increment of the angle strain.

The most practically important conclusion in the work of P.W. Rowe is a quantitative assessment of the change in shear strength when dilatancy of the dispersed medium is limited. Obviously, with free volumetric strains, soil compaction or loosening will be observed during shear. But in real foundations the volumetric strain of the soil is most often limited, and instead of increase in volumetric strains an increase in the mean stress is observed, which leads to an increase in the shear strength. Instead of fixed value of the internal friction angle Rowe proposes to use its variable value depending on the degree of development of dilatancy (therefore, also the degree of approaching a state of limit equilibrium).

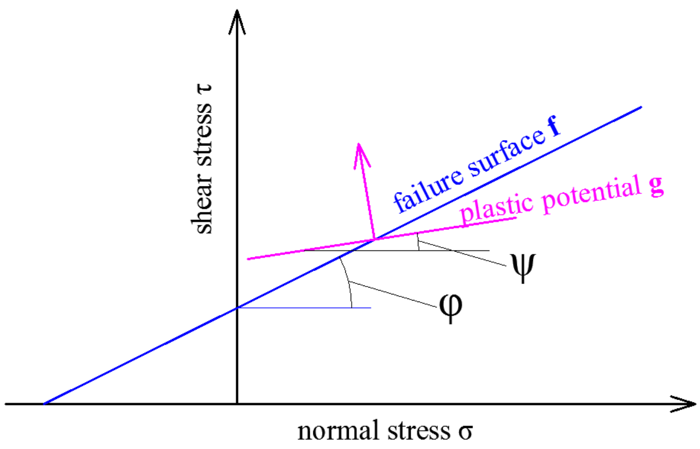
Based on extensive laboratory experiments, M.D. Bolton obtained correlations between the peak and critical angles of internal friction and also the degree of density $I_D$. It was shown that for dense soils the angle of dilatancy can reach 15°, which makes it possible to increase the strength substantially. These studies show that the angle of dilatancy is the complex parameter, that depends on the physical characteristics (dry bulk density), strain characteristics (volumetric compressibility) and strength characteristics (shear strength). This complicates a laboratory assessment, since the value of the parameter varies depending on the determination method and mode.

As the generalization of foreign studies of dilatancy, the work by Vermeer and De Borst can be considered. It contains the most important conclusions, obtained by researchers for 50 years.

Russian specialists have also studied the phenomenon of dilatancy. The results of basic researches of the dilatancy were given in the works of S.V. Bakushev, G.G. Boldyrev, M.V. Malyshev, D.Yu. Sobolevsky. They considered the physical and mechanical meaning of this parameter, modifications of the plasticity theory.

It should be noted that the absolute majority of studies of dilatancy, as well as methods for its quantitative assessment, consider sands. This is due to both the large size of the particles and their shape. The characteristic platy shape of the particles of clay soils does not lead to the additional volumetric strains when sheared because of the higher porosity and the possibility of relative rotation of separate particles. As a result, volumetric deformations when sheared are so insignificant that their measurement in laboratory studies is extremely difficult. This explains the recommendations of researchers to take the value of the angle of dilatancy equal to zero for normally compacted soils. At the same time, in overconsolidated soils, aggregated particles can be formed. These aggregates can be hard enough to implement the dilatancy mechanism.

The angle of dilatancy is used in the plasticity theory to consider volumetric strain during plastic flow. This solution was proposed by Drucker and Prager in their work on the non-associated flow law for granular soils. In addition to the yield surface determined by the Mohr–Coulomb criterion, a second surface is introduced – the plastic potential, the slope of which is determined by the angle of dilatancy. If the angle of dilatancy $\psi$ is equal to the angle of internal friction $\varphi$, the yield surface is simultaneously considered as the potential surface ($g = f$), i.e., the associated flow law applies. For most soils in this case, unrealistically large volumetric strain occurs, and the real angle of dilatancy is usually several times less than the angle of internal friction. When $\psi < \varphi$, the model uses a non-associated flow law, and when $\psi = 0$ there will be no volumetric strain during shear (the plastic potential becomes a horizontal line).



All these positions are included in modern mathematical models that use the Mohr–Coulomb failure criterion: the Hardening Soil model and its derivatives (Hardening Soil, Hardening Soil Small-strain), the Soft Soil model and its derivatives (Soft Soil, Soft Soil Creep). Thus, in numerical modelling, use of the real value of the angle of dilatancy makes it possible to model additional shear strength for soils with high density – in the foundations of loaded structures, embankments and levees, which leads to a noticeable economic effect and a more complete use of the bearing capacity of the soil.

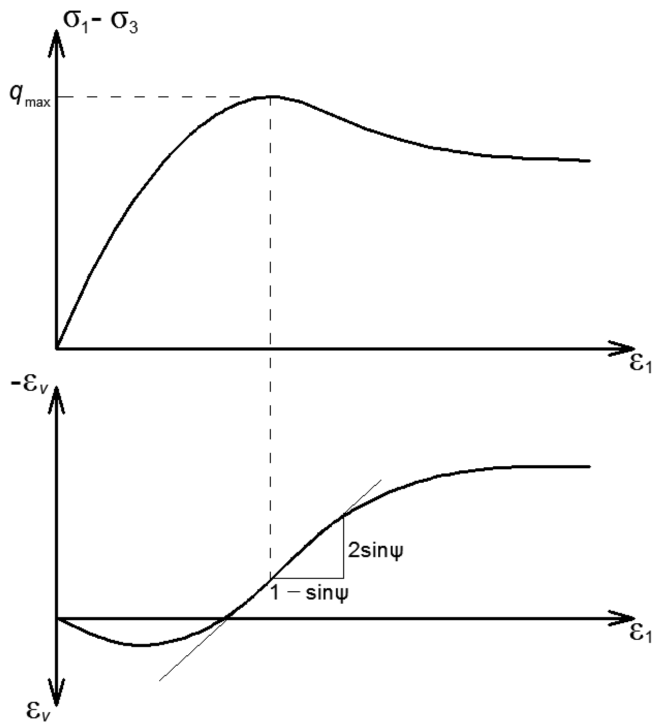
By physical meaning in the practice of engineering surveys, the angle of dilatancy can be determined in any instrument, which makes it possible to measure the volumetric strains of soils with substantial development of shearing strains, in particular in the instruments for triaxial compression tests.

During deviatoric loading of the sample, a change in volume due to dilatancy is recorded. The maximum dilatancy usually appears at the peak strength of the sample. Therefore, to determine it, it is necessary to select the range of volumetric and shear strains close to the maximum value of the deviator. For triaxial compression conditions P.A. Vermeer and R. De Borst proposed a formula for determining the angle of dilatancy:

$$\psi = \arcsin\left(\frac{\Delta\varepsilon_v}{\Delta\varepsilon_v - 2\Delta\varepsilon_1}\right).$$

As in other cases, in this formula it is necessary to observe the rule of signs: the increment of relative vertical strains is negative, and the increment of relative volumetric strains is positive when dilatancy and negative when contraction.

Graphically, the determination of the angle of dilatancy is performed according to the plot of volumetric strains versus axial strains at the point corresponding to the peak strength or in the area of maximum slope.



The plot clearly shows that at the initial stage of the application of the deviatoric load, the sample is additionally compacted (due to the increasing mean stress), but subsequently volumetric strains are stabilized, and in dilating soils volumetric expansion can be observed. Obviously, such a representation makes sense only for testing in accordance with the CD scheme, since the CU and UU schemes assume constant volumetric strains.

The advantage of this method is its high sensitivity: determination of the angle of dilatancy is possible even for sandy loam and clay. Furthermore, most often these tests are carried out, when the use of numerical calculation methods is assumed and the determination of the angle of dilatancy can be performed according to the results of the same testing, without an increase in the estimated cost and the number of necessary samples, since no separate experiment is required.

OOO "GEOTEK" proposes the automated testing system for axisymmetrical CD triaxial tests of the soil samples with the possibility of high-precision measurement of volumetric strains when sheared. The system includes the triaxial compression cells and also the necessary equipment for implementation of vertical force action, cell and back pressure management. The tests are carried out in an automated mode with the control of all test parameters in real time.

More detailed technical information can be given by the company's specialists or found on the official website https://npp-geotek.com/en/.